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Fuzzy Sets and Fuzzy Logic

Abstract

The following is a brief overview of the major properties and features of fuzzy sets and fuzzy logic and an investigation of the major techniques for the implementation of fuzzy systems.

How to Use this Presentation

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- This is not a traditional presentation. The slides have more text as they are not intended to be used in a lecture environment
- You should read the slides and consider the questions posed. You may wish to record your answers and discuss them
- A Spreadsheet is also provided for you to experiment with

Human Classification and Reasoning: Generalisation

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Normal human reasoning relies heavily on the use of broad classifications of objects, processes and conditions.

These are not rigorously defined, but provide generalised rules that may be applied to thoughts and objects that have the same basic features.

The ability to generalise is important, it allows existing knowledge to be applied to new situations. Without the ability to generalise, each problem encountered would require a unique solution.

A Blue Marble

Human Reasoning and Language

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The classifications used in language are often *vague.* What does a term like *Large*, *Fast*, *Hot* or even *Very Hot* mean?

Fuzzy logic allows this vagueness to be captured and applied in our solutions to problems.

Descriptions of solutions to difficult problem can be provided by experts and then encoded using fuzzy logic.

Fuzzy logic provides us with a way of capturing these *Linguistic Variables*

Partial Truth

In Artificial Intelligence or Control Engineering the action taken might depend on the value of a continuous variable.

Each action will be the result of the variables falling within a specific range.

With traditional, binary logic means that the transition between actions may be abrupt.

This may not be ideal.

BMI – Body Mass Index

BMI is a tool used to determine if a person is at risk of conditions such as Type 2 Diabetes or heart disease

Knowing their BMI helps a patient decide what action they should take to improve their health

As a measure it is determined from both their weight and height:

$$BMI = weight(kg)/height(m)^2$$

BMI – Classification

BMI is classified into 5 bands:

- < 18.5 – underweight.
- between 18.5 and 24.9 – normal
- between 25 and 29.9 – overweight
- between 30 and 39.9 – obese
- > 40 – morbidly obese

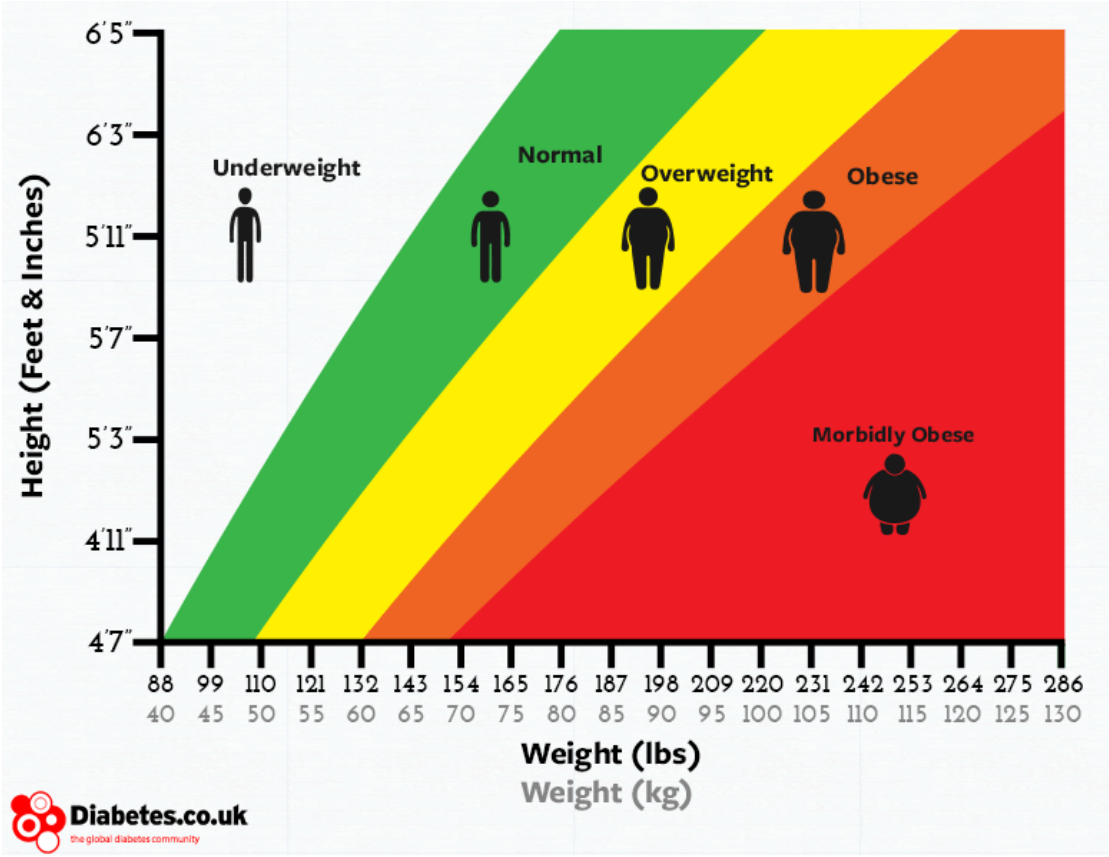


Image reproduced from https://www.diabetes.co.uk/bmi.html

BMI – What to do

A patient calculates their BMI as 24.9

They discover they are on the boundary between being a normal weight and over weight.

What should they do?

- As a normal weight they may decide to take no action

What if they had calculated their BMI as exactly 25?

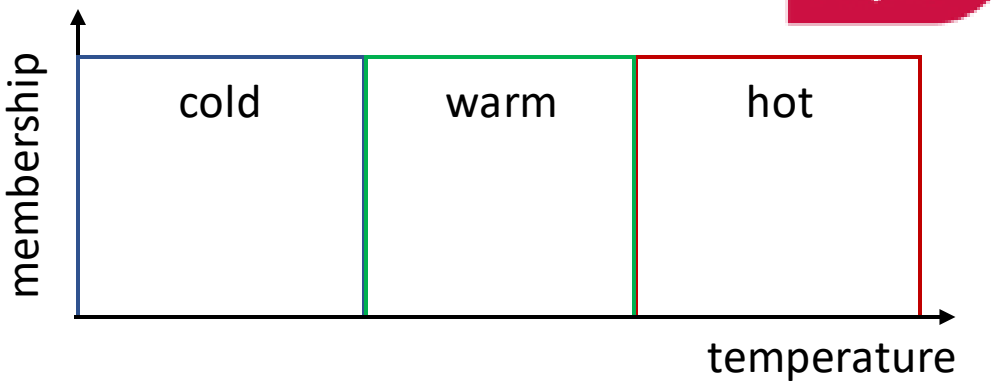
- The difference in weight is minor. Should their behaviour really be that different?

What if they had calculated their BMI as 29.8?

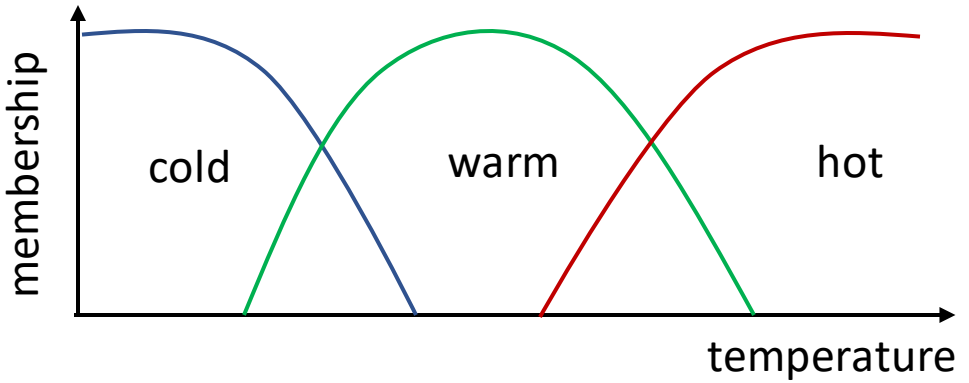
Fuzzy Sets

Fuzzy sets allow provide the mechanism to capture linguistic variables and record vagueness

They do this by allowing non-binary membership of a set



Crisp sets – Binary Membership



Fuzzy sets – continuous values of membership

Membership Functions

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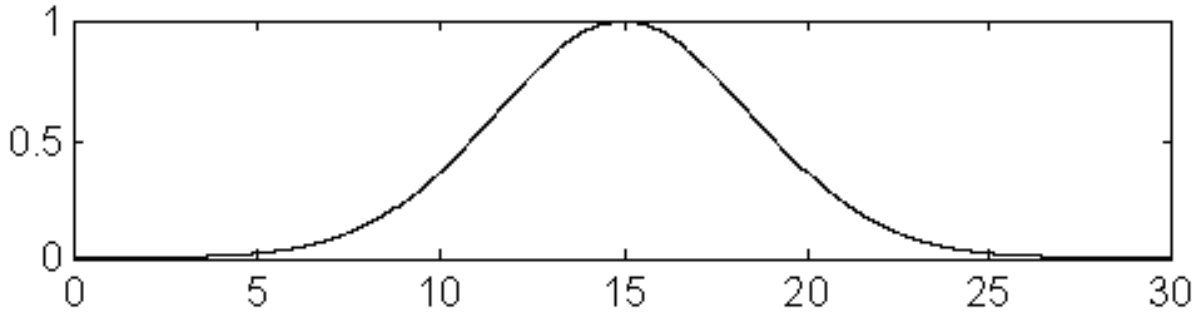
- The membership function describes the membership of each value of a variable for a fuzzy set
- Membership functions may be of any form, however there are standard forms such as trapezoidal and Gaussian functions
- Triangular and trapezoidal functions are often employed as they are less computationally demanding. They may be used as approximations to more complex functions
- A crisp set has a rectangular membership function

Gaussian Membership Function

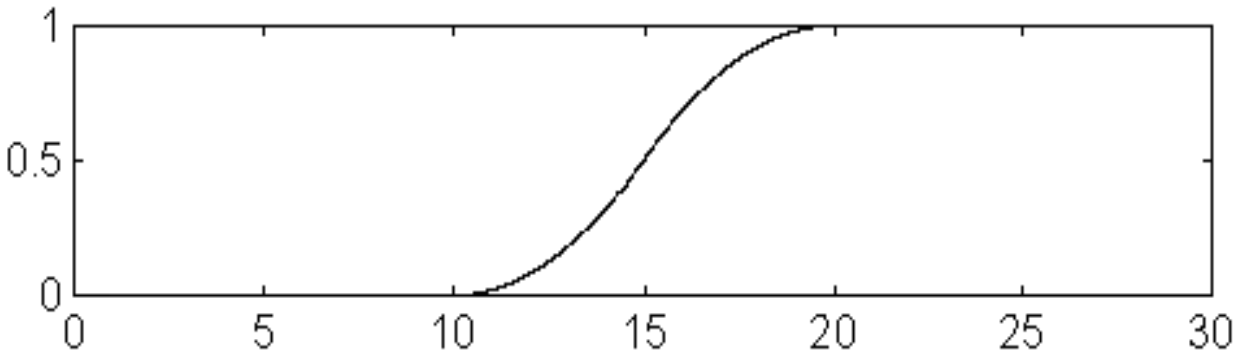


G(x,15,5)

$$G(x, \mu, \sigma) = e^{\frac{-(x-\mu)^2}{\sigma^2}}$$

The Gaussian curve is extensively used. Although it is more complex than the trapezoidal function, it preserves information

S and Z curves



S(x,10,15,20)

$$S(x,a,b,c) = 0; x \leq a$$

$$= 2\left(\frac{x-a}{c-a}\right)^2; a < x \leq b$$

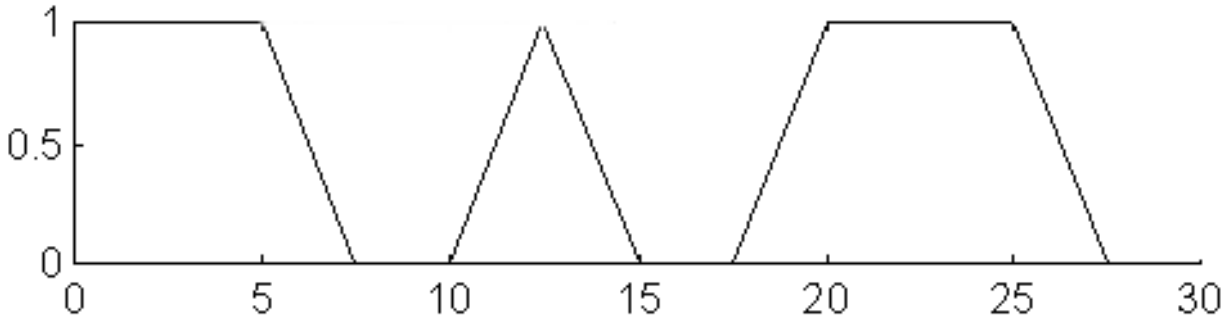
$$= 1 - 2\left(\frac{x-c}{c-a}\right)^2; b < x \leq c$$

$$= 1; x > c$$

At the ends of a variables range (*Universe of Discourse*) an open-ended function is appropriate.

The Z function is the complement of S

Piecewise Membership Functions



Piecewise Z, triangular and Trapezoidal membership function

The trapezoidal, triangular and piecewise S and Z functions allow for easy calculation of membership and are often sufficient. Issues can occur due to *fuzzy aliasing*

Membership Function Activities

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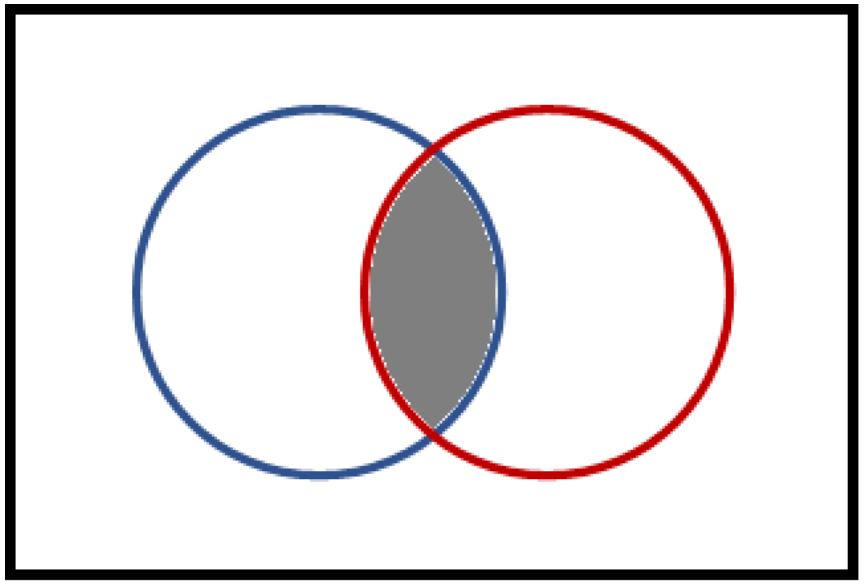
What are the membership functions used in the provided spreadsheet example?

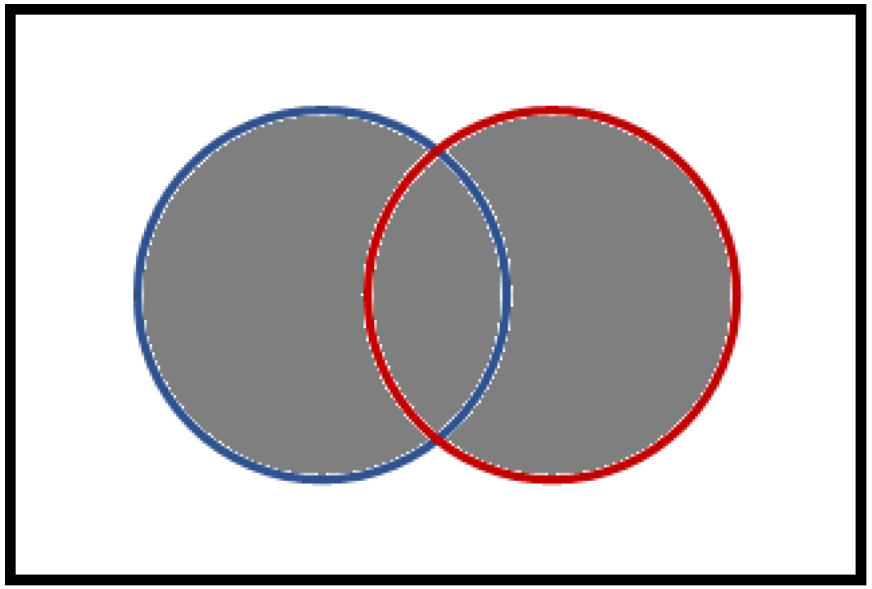
Could you change the spreadsheet to use the Gaussian and S and Z functions (keeping the set boundaries roughly the same)?

Set Operations

All standard set operations may be performed on fuzzy sets, but there may be alternative methods of achieving each which produce different results.



Set A AND Set B

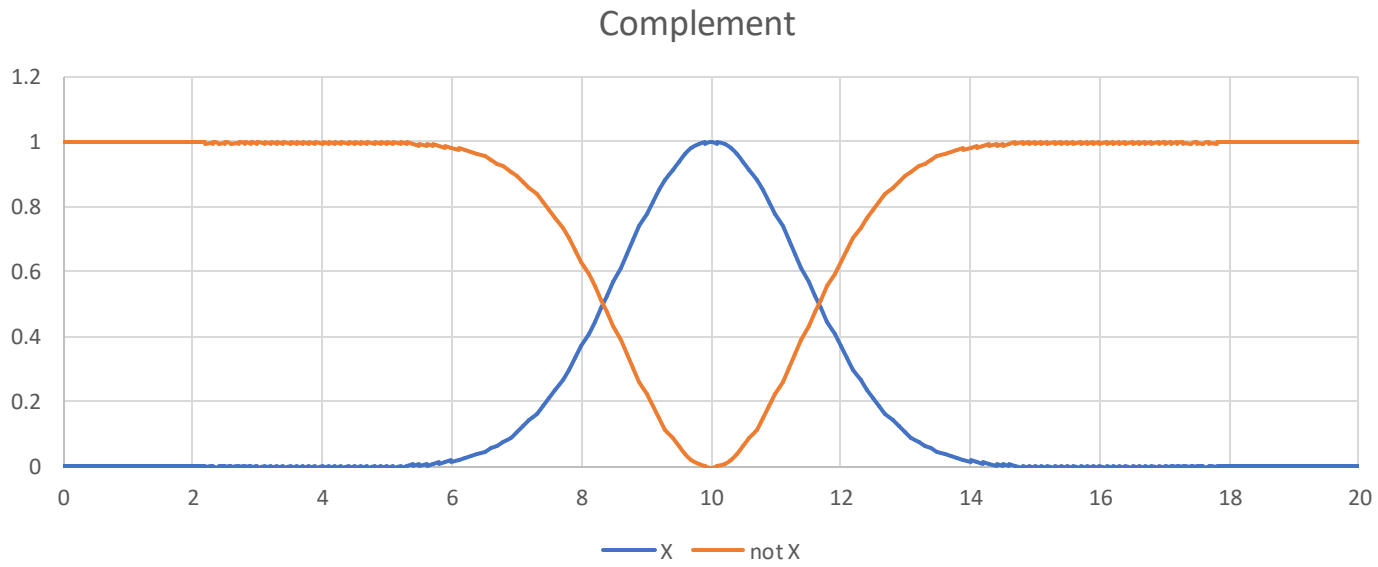


Set A OR Set B

Complement (Not)

The complement of a fuzzy set is found simply:

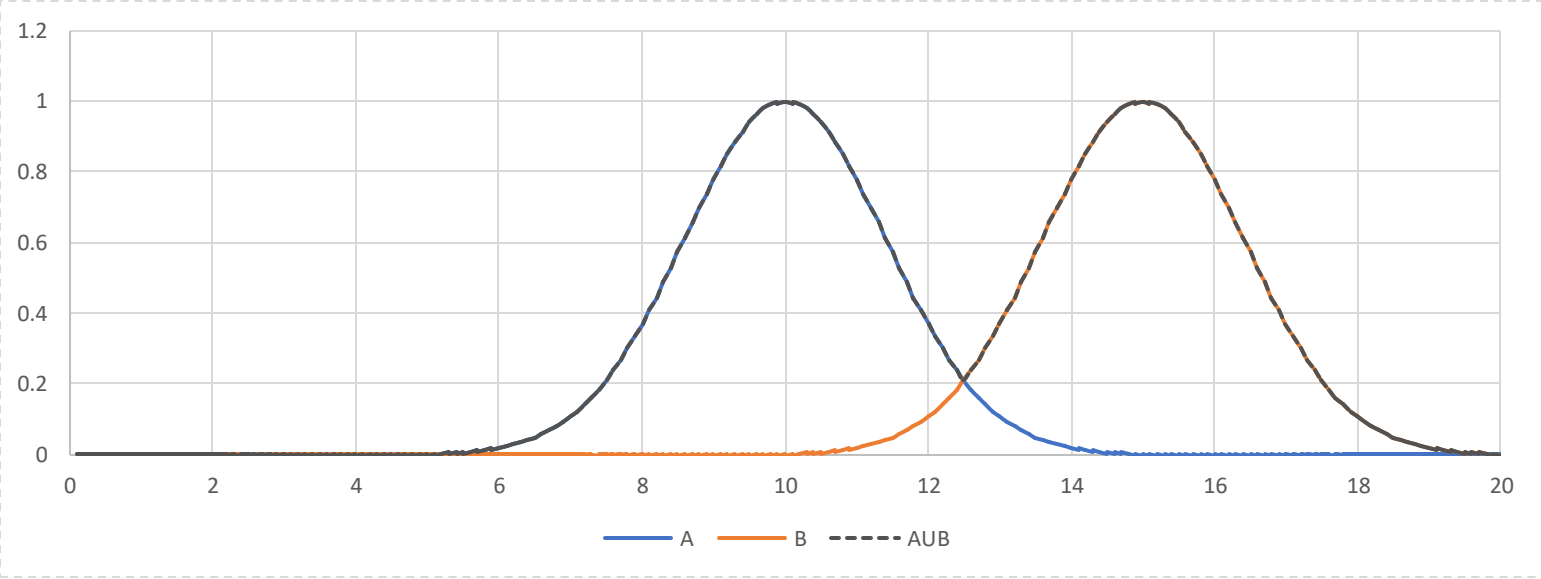
$$\mu_{setA}(x)' = 1 - \mu_{setA}(x)$$



Union (Or)

The simplest method of finding the Union of two fuzzy sets is to find the maximum value of their memberships

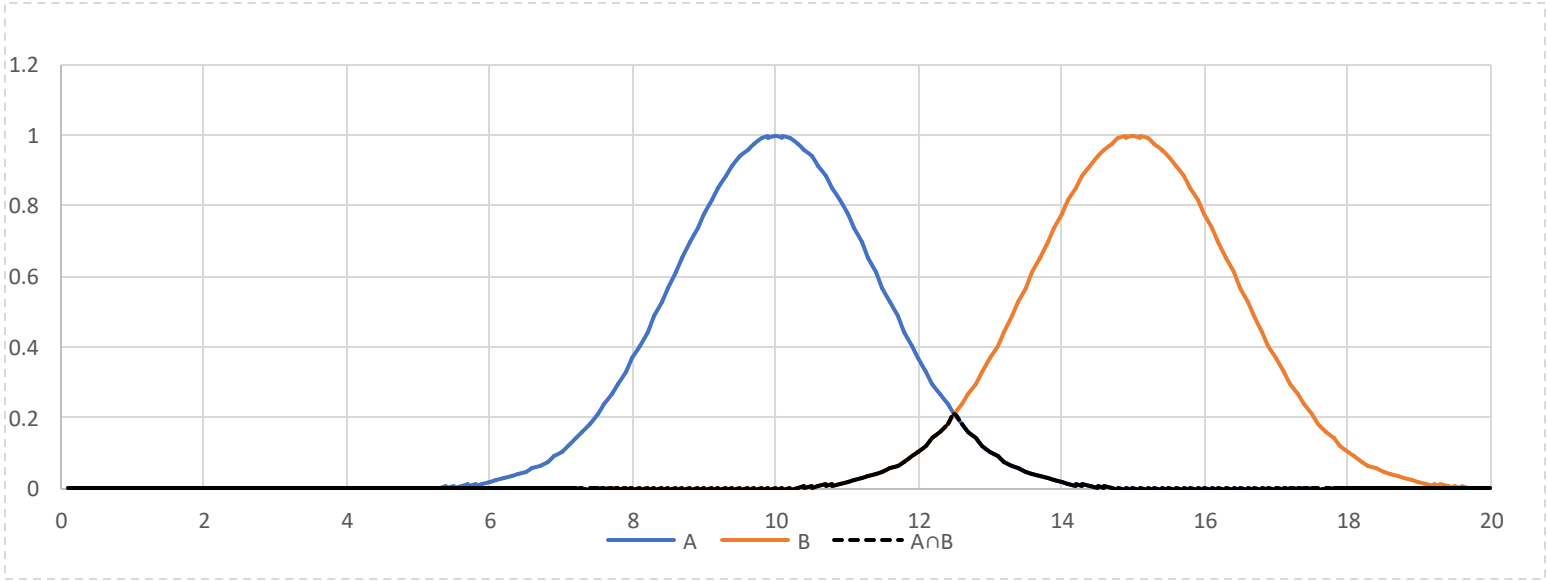
$$\mu_{setA} \cup \mu_{setB} = MAX[\mu_{setA}, \mu_{setB}]$$



Intersection (And)

The intersection of two fuzzy sets can be found by taking the minimum of the sets

$$\mu_{setA} \cap \mu_{setB} = MIN[\mu_{setA}, \mu_{setB}]$$



Applying Fuzzy Operators

Fuzzy sets and operators can be used to generate rules:

$$If\ (variableX = setA)\ AND\ (variableY = setB)\ THEN\ (setC)$$

$$If\ (variableX = setA)\ OR\ (variableY = setB)\ THEN\ (setD)$$

For each rule a value or *firing weight* is calculated by applying the operators. These firing weights are used to determine what action to take.

Note that the *AND* operator is the most commonly used in fuzzy inference.

Determining Output

When the rule firing weights have been calculated the system must determine what action to take or a crisp output.

Mamdani inferencing uses fuzzy output sets associated with each rule.

In a Sugeno system the firing weight isn't applied to a fuzzy set but to the coefficients of a linear equation. For a zero order Sugeno system that is simply:

$$If\ x = a\ AND\ y = b\ THEN\ output = z$$

The general case:

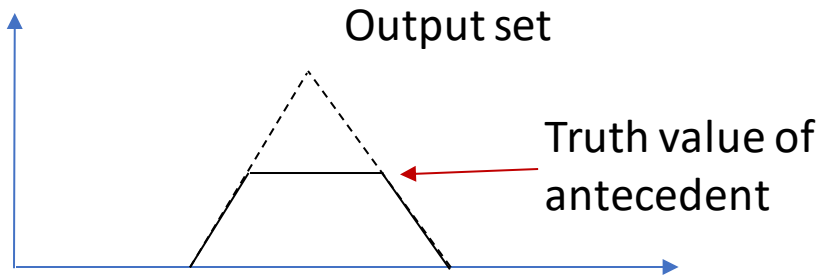
$$If\ x = a\ AND\ y = b\ THEN\ output = p.x + q.y + r; \qquad p, q, r\ constant$$

Applying weights to output Fuzzy Sets

The truth value of a rule; its firing weight must be transferred to the output set. There are two simple ways of doing this:

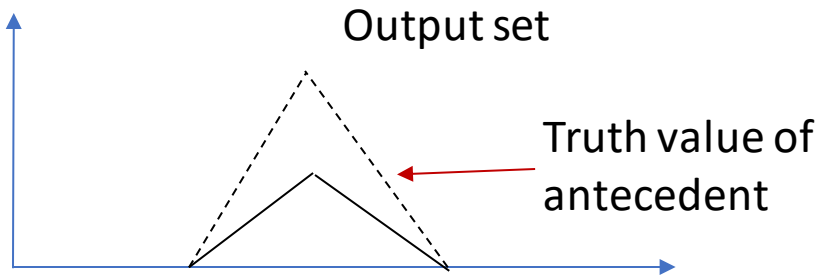
Minimum:

The set is truncated



Product:

The set is scaled



Aggregation

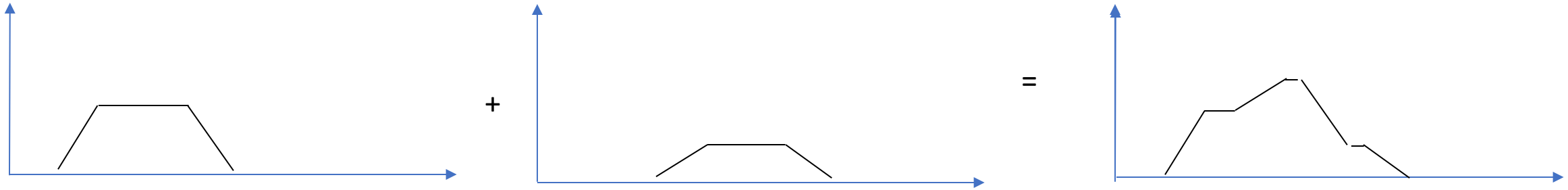
The outputs of the various rules in the rule base must be combined. There are several approaches:

MIN-MAX

Output sets of rules are truncated by firing weight. Following this only the set with the rule with the maximum output is used for defuzzification

Additive

The sum of the output sets is found and sent for defuzzification



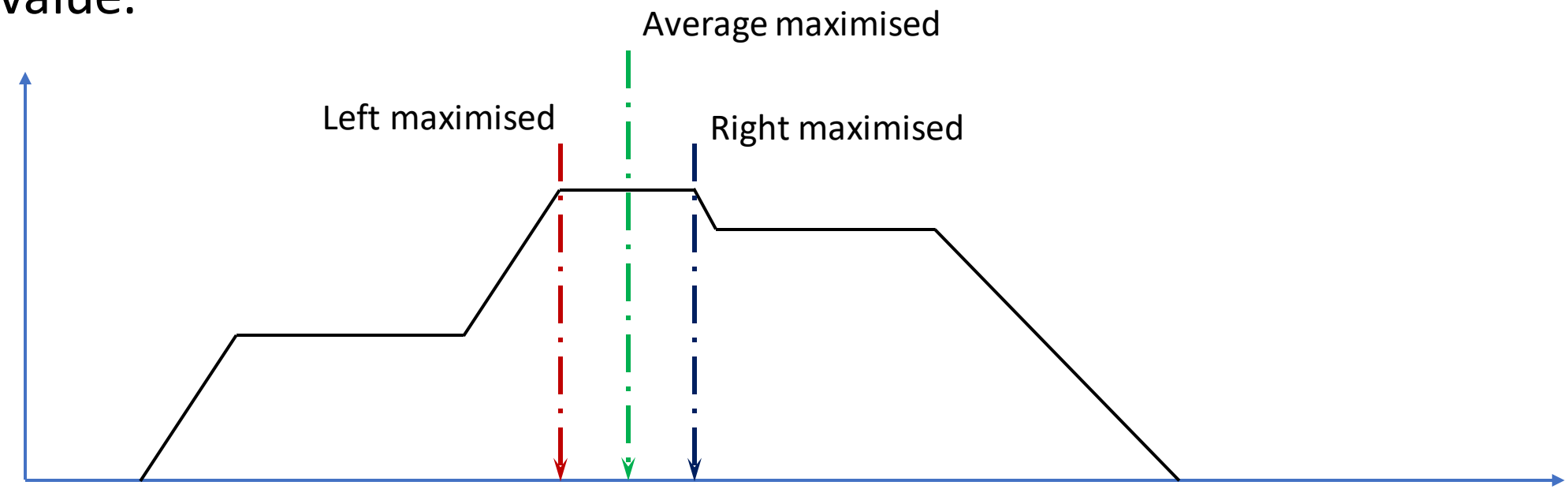
Defuzzification

Defuzzification is the process of taking the fuzzy output and deriving a single value. This value may be the control signal to a motor for instance.

Again there are different techniques

Maximised

- The crisp output is derived by determining the output value which represents the maximum point in the aggregated fuzzy outputs. Where a plateau exists a choice is made as to where to take this value.



Centroid

Centroid defuzzification is commonly used. It produces smooth changes in output which is often important in control systems.

The centroid may be calculated or a sum of discrete points used as an approximation:

$$Output = \frac{\int x\mu(x)dx}{\int \mu(x)} \quad \text{Or} \quad Output = \frac{\sum_{i=1}^{N} x_i \mu(x_i)}{\sum_{i=1}^{N} \mu(x_i)}$$

Sugeno

Defuzzification of zero order Sugeno systems requires calculating the weighted average of the rule singleton values, where the weight is the firing weight for that rule.

Sugeno inferencing is often preferred as calculations are easier, and unlike the centroid method of defuzzification the final output can exploit the full range of the output variable.

Fuzzy Algorithms

Fuzzy logic isn't just used in such rule-based systems. Fuzzy algorithms may also be used. These are an ordered list of instructions like any other algorithm, but instructions may operate on fuzzy sets.

As with standard algorithms, the statements fall into three main types: *assignment*, *conditional* and *unconditional*.

Conditional statements are resolved either by the defuzzification of the tested variable, or the parallel realisation of the branches.

Summing Up

- Fuzzy sets provide a simple mechanism for the coding of human knowledge
- Fuzzy logic can code Linguistic Variables and Vagueness (Not probability).
- Fuzzy Logic has become popular in applications where mathematical models are highly complex, yet there is an intuitive solution that may easily be described.
- Fuzzy logic has been successfully used in large numbers of real applications, including electronic devices such as cameras and microwave ovens.
- A major advantage is the ability to increase the MIQ [Machine Intelligence Quotient] of devices cheaply and relatively simply. This simplicity is the direct result of the use of linguistic variables.

The spreadsheet - instructions

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- Inputs can be typed into cells A4 and A12
- Output is given in cell O42
- The charts show the input membership functions and the point where the input occurs in the universe of discourse.
- The spreadsheet is fully editable
- It is provided for you to experiment with

The spreadsheet – some things to try

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A spreadsheet is provided that implements a simple Fuzzy control system with two variables.

- Try changing the values and watching how the outputs change
- Make sure you understand all the calculations,. A number of “half way house” calculations are done to make it easier to follow
- Can you change the number of sets on either of the inputs?
- Can you change the sets from trapezoidal to Gaussian (and S and Z functions)?
- How would you go about changing the inference from Zero order Sugeno to Mamdani with centroid aggregation?

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